Soft Subdivision Motion Planning for Complex Planar Robots^

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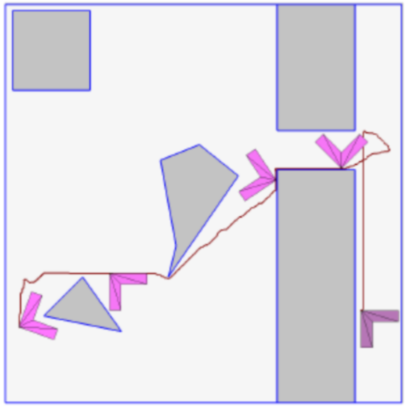
Motion Planning

- A central problem in robotics

– There is a fixed rigid robot: $R_0 \subseteq R^k$ (k = 2,3)

– Configuration: pos. & orientation of a point p in R_0



INPUT : (α, β, Ω)

- Start and Goal configurations α, β
- Polyhedral obstacle set $\Omega \subseteq R^k$ (k = 2,3)

OUTPUT:

- A path from α to β avoiding all obstacles in Ω, if it exists.
- Else report "NO PATH".

State of the Art

(A) Exact Methods

+ Strong theoretical guarantees
- High complexity

e.g., roadmap is single exponential time [Canny 93]

basic path planning is semi-algebraic (book of [Basu-Pollack-Roy])

- Complex to implement & expensive to compute

(rarely implemented and not practical)

(B) Subdivision Methods (e.g., [Zhu-Latombe91], [Zhang et al 08])

Fairly popular but ``does not scale''

Often degenerate into "grid method"

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State of the Art (cont.)

(C) Sampling Methods

* Probabilistic Road Map (PRM) [Kravraki 96];
  many variants: EST, RRT, SRT, etc.

* Dominate the field in the last 2 decades.

Major Issue: Halting Problem ("Narrow Passage" problem) ---
Don't know how to halt when there is no path (except for artificial cut-off)

• Some subdivision work (e.g., [Zhang et al 08]) can detect non-existence of paths, but cannot guarantee to always detect that (sol. is partial).

State of the Art (cont.)

Resolution-Exact Algorithms

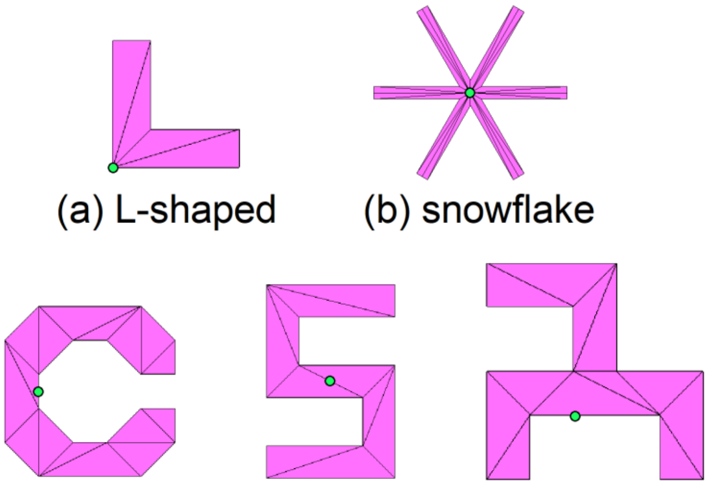
- We initiated in [Wang-Chiang-Yap SoCG13], [Yap 13]
- Use subdivision and soft predicates --- Soft Subdivion Search (SSS)
- Avoid exact computation, easy to implement correctly, run fast, always halt, with theoretical guarantees (see paper for details).

- Further extended for 2-link planar robot with 4 degrees of freedom (4 DOFs) [Luo-Chiang-Yap WAFR14], [Chee-Luo-Hsu WAFR16], 5-DOF 3D robots [Hsu-Chiang-Yap 18].

- In this paper, we work on 2D complex robot under this framework.

New Results: SSS for Complex Robots

- 2D rigid complex robots with arbitrary complexity (*m*-sided polygon, m>=5).



(a) L-shaped (b) snowflake (c) C-shaped (d) S-shaped (e) 3-legged

- Use triangulation.
- Our previous [SoCG 13] method for triangle robot does not work since the triangles in a complex robot must share a common origin (rotation center).

Review of SSS: Resolution Exactness

- An resolution-exact planner takes an extra input parameter $\varepsilon > 0$. It always halts and outputs either a path or NO-PATH. The output satisfies:

There is an accuracy constant $K > 1$, s.t.

– If exists a path of clearance $K\varepsilon$, it must output a path;

– If there is no path of clearance ε/K, it must output NO-PATH.

– Indeterminacy allowed (small price for avoiding exact computation)

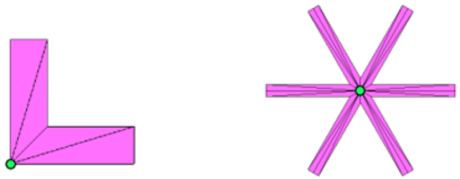
Review of SSS: Search Framework

- Maintain a subdivision tree *T* rooted at box B_0 (input domain)
- Each internal node is a box *B*, which is split into 2^i ($1 \le i \le d$) congruent subboxes (T/R-split: see later)
- Each box *B* is classified as
  free (each $t \in B$ is a free configuration),
  stuck (each $t \in B$ is in the exterior of the free space), or
  mixed (otherwise).

- We maintain connected components of free boxes via a Union-Find data structure ($\alpha, \beta \in$ same component → path found)
- Priority Queue *Q* of mixed boxes to be expanded later

Star-Shaped Robots

A star-shaped region *R:* there exists a point *A* ∈ *R* s.t.
A can "see" every point in *R*.
We call *A* a **center** of *R*.

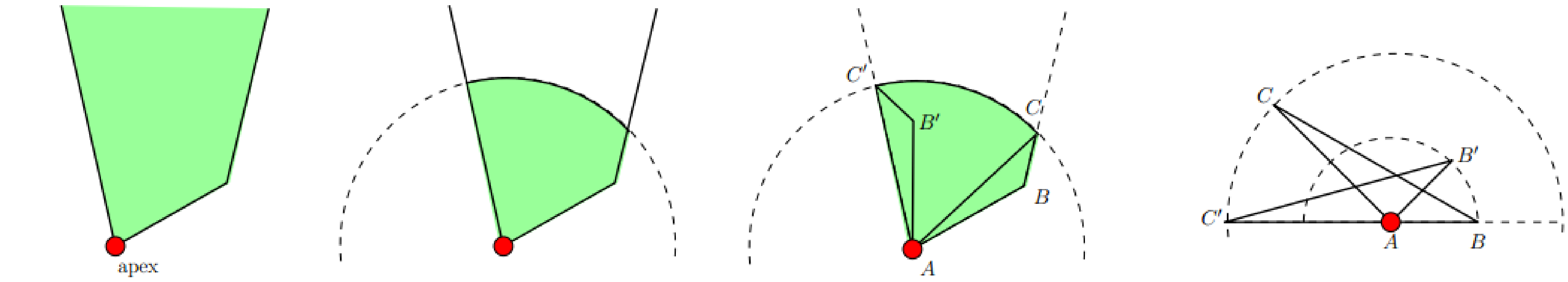


When a robot R_0 is star-shaped, we decompose R_0 into a set of triangles that share a **common vertex** at a center *A*.

We need a predicate that can easily classify boxes B as free/stuck/mixed.

Star-Shaped Robots

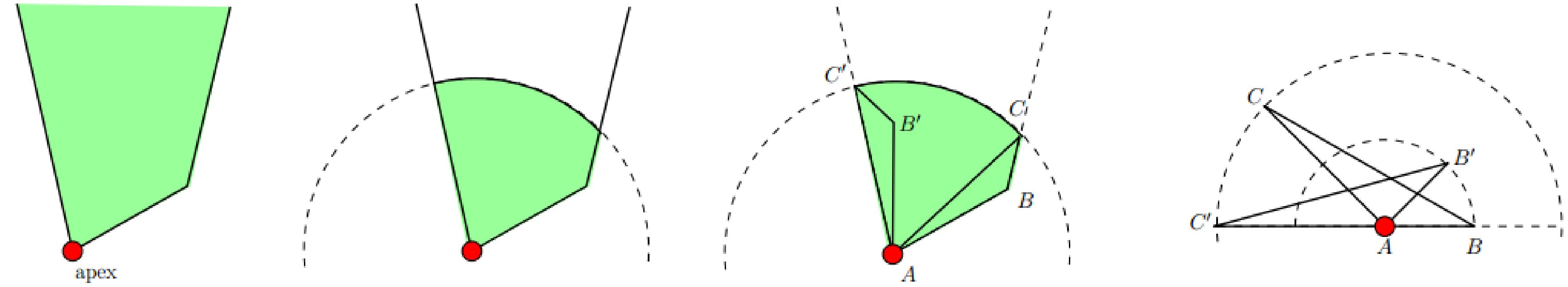
- Triangular Set: $T = H_1 \cap H_2 \cap H_3$ (intersection of three half-spaces) T can be bounded (triangle) or unbounded (Figure (a) below)
- Apex: distinguished vertex (red)
- Truncated Triangular Set (TTS): $TTS = T \cap D = H_1 \cap H_2 \cap H_3 \cap D$ T intersects with a disc D centered at A (Figure (b) below)



(a) triangular set (unbounded case) (b) truncated triangular set (c) swept area by a nice triangle (d) sweeping $[ABC]$ to $[AB'C']$

Star-Shaped Robots

- Angular Range: $\Theta = [\alpha, \beta]$
- Swept area $T_0[\Theta] = T_0[\alpha, \beta]$ for triangle T_0 (A,B,C where A is the apex)
- Nice swept area (Figure (c) below) and not nice (Figure (d) below)
- Nice triangle: b ≥ $\pi / 2 = 90°$
  (where a,b,c are the angles of vertices A,B,C resp.)



(a) triangular set (unbounded case)
(b) truncated triangular set
(c) swept area by a nice triangle
(d) sweeping $[ABC]$ to $[AB'C']$

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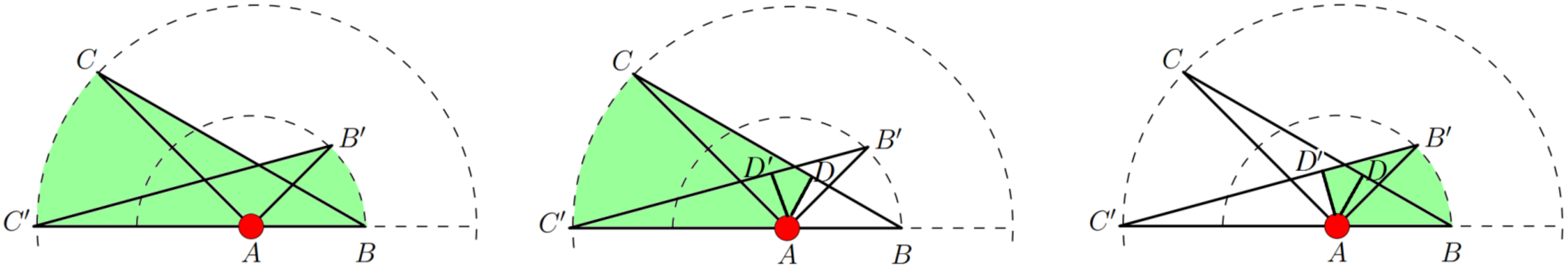
- Nice triangle: $b \geq \pi / 2 = 90^\circ$
  (where a,b,c are the angles of vertices A,B,C resp.)

- Lemma 1: T is nice $\Leftrightarrow$ Footprints $T\,[0, \alpha]$ and $T\,[-\alpha, 0]$ are TTS for all α where $0 < \alpha < \pi - a$

Star-Shaped Robots

- Lemma 2: A star-shaped robot R_0 (an n-gon) can be decomposed into an essentially disjoint union of at most 2*n* nice triangles sharing apex A



Star-Shaped Robots

- Complex Predicates: $R_0 = \cup_{j=1}^{m} T_j$

$$\tilde{C}(B) = \begin{cases} \text{FREE} & \text{if each } \tilde{C}_j(B) \text{ is FREE} \\ \text{STUCK} & \text{if some } \tilde{C}_j(B) \text{ is STUCK} \\ \text{MIXED} & \text{otherwise} \end{cases}$$

- T/R Splitting
- Do *translational* (T) splitting only and keep the rotational component full until the box width is ≤ ε (ε-small) --- top: quad tree
- Do *rotational* (R) splitting if the box is ε-small --- bottom: binary tree

Feature Set of a Box

- Each box B: we use its feature set $\tilde{\phi}(B)$ to classify B as free/stuck/mixed.
- Obstacles Ω: polygonal set $\Omega \subseteq R^2$
- Look at boundary of Ω. Feature *f* : corner or edge of boundary of Ω
- Feature set $\tilde{\phi}(B)$: contains all *f* that are potentially in conflict with robot R_0 when its configuration is in B.
- As we split a box into subboxes, the feature sets become smaller.
- **Classification:**

  $\tilde{\phi}(B)$ is non-empty: B is **mixed**.

  $\tilde{\phi}(B)$ is empty: B is in no conflict with obstacle boundary ➔ B is **free** or **stuck** (use parent feature set to decide)

Feature Sets for Star-Shaped Robots (I)

Soft Predicates for classification

- When B is a T-split box (we only split its translational box; quad-tree part)
  Its **features set** $\widetilde{\boldsymbol{\phi}}(\boldsymbol{B})$ comprises those features f such that

  $$\mathrm{Sep}(m_B, f) \le r_B + r_0$$

  where m_B and r_B are the midpoint and radius of translational box of B,
  r_0 is the radiuis of robot R_0

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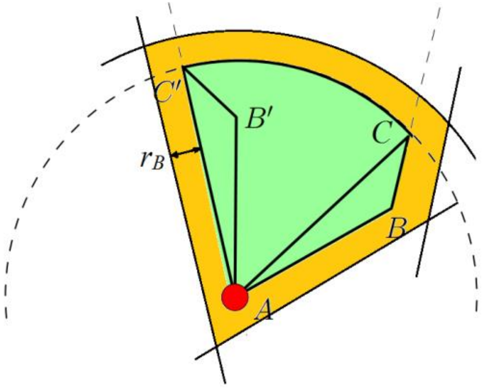
Feature Sets for Star-Shaped Robots (II)

When B is an R-split box (we only split its rotational box; binary-tree part)

- $\widetilde{\phi}(B)$: a collection of $\widetilde{\phi}_j(B)$ for each nice triangle T_j
- *TTS_j : apex is at the box center m_B*
- Let Q be a shape and s be a real number, **s-expansion** of Q is defined as the Minkowski sum of Q with the $Disc(s)$ of radius s centered at the origin



- $\widetilde{\phi}_j(B)$ comprises those features f satisfying
  (1) $Sep(m_B, f) \le r_B + r_j$ (2) f also intersects the r_B-expansion of TTS_j (yellow: super set)

  Condition (2) can be easily checked

General Complex Robots

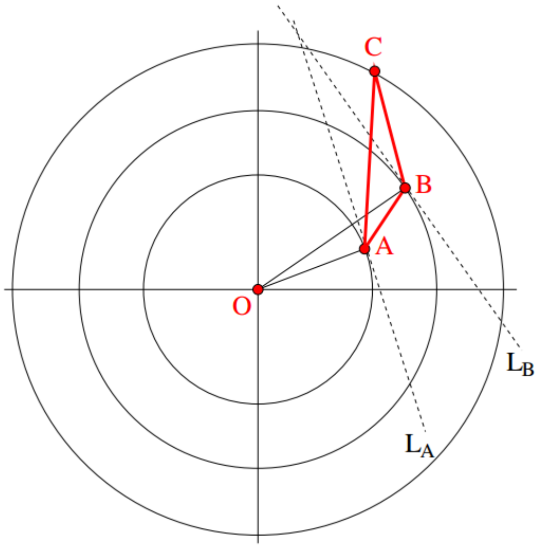
- R_0 is a general polygon, we can still decompose R_0 into a set of triangles T_j ($j = 1, . . . , m$)
- The rotation of these triangles are relative to a fixed point O
- We will define T_j to be "nice relative to a point O"

General Complex Robots

- Let $T = [A, B, C]$, O be the origin (outside of T)
- Let $0 \leq ||A|| \leq ||B|| \leq ||C||$ where $||A||$ is the Euclidean norm of a vector A

- **We say T is nice if**

$$\langle A, B - A\rangle \geq 0,$$
$$\text{and } \langle A, C - A\rangle \geq 0,$$
$$\text{and } \langle B, C - B\rangle \geq 0.$$

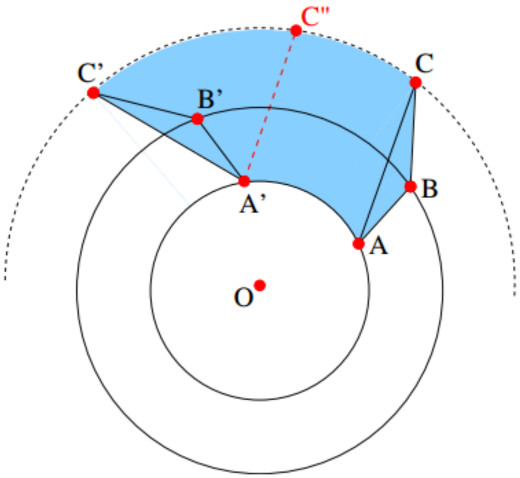


General Complex Robots

- If T is a nice triangle, $T\,[\alpha, \beta]$ is called a nicely swept set (NSS).

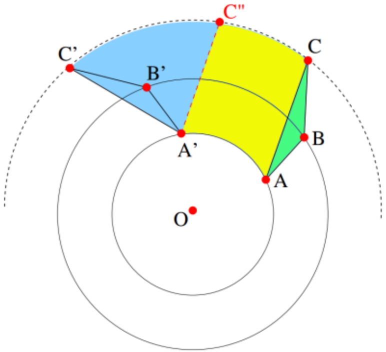
We want an easy way to detect the intersection between an s-expansion of NSS and any feature (point or edge)

- We define a subset of R^2 as a:
  - 0-basic shape: half-space, a disc or complement of a disc
  - 1-basic shape: finite intersection of 0-basic shapes
  - 2-basic shape: finite union of 1-basic shapes

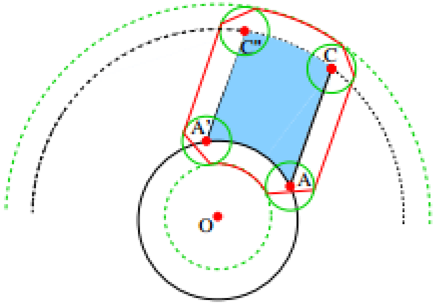


General Complex Robots

- E.g. 1-basic shapes:
  - Triangles (ABC)
  - Sectors (A'C'C")
  - Truncated strips (ACC''A' --- shown in yellow)
- The s-expansion of a *sector / truncated strip / triangle* is 2-basic.



Theorem: Let $T\,[\alpha, \beta]$ be a nicely swept set where $[\alpha, \beta]$ has width $\leq \pi / 2$. It can be decomposed into a triangle, a sector and a truncated strip**.** The s-expansion of $T\,[\alpha, \beta]$ has a basic decomposition into 2-basic shapes.

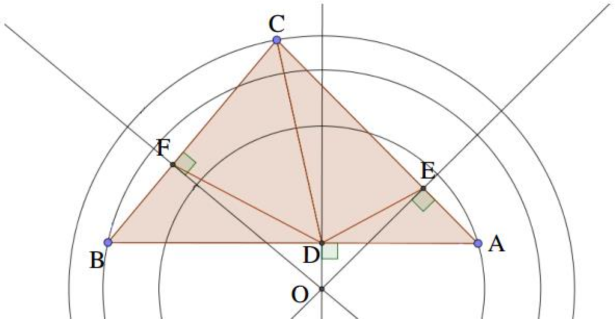


Testing intersection of 2-basic shapes with any feature is O(1).

General Complex Robots

- Partitioning an n-gon into Nice Triangles
  − First triangulate into n-2 triangles
  − For the one contains the origin O, split into 6 nice triangles using the star-shaped technique
  − **Lemma:** If T is an arbitrary triangle and O is exterior to T , then we can partition T into at most 4 nice triangles.

Theorem: Given any triangulation of P into n - 2 triangles, we can refine it into ≤ 4n - 6 nice triangles.

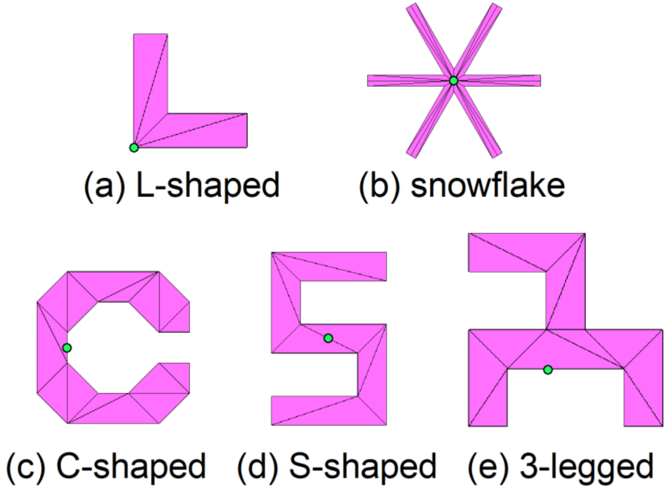


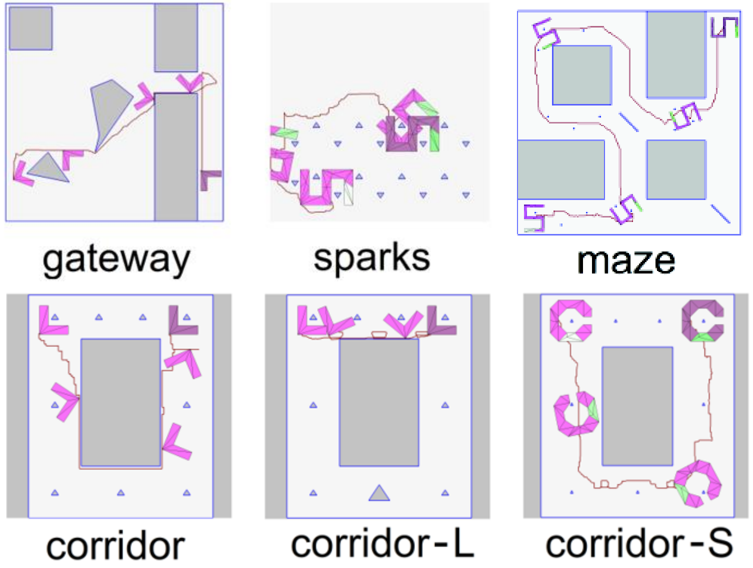
General Complex Robots

- Soft Predicates: similar to the technique for star-shaped robots
- $\widetilde{\phi}_j(B)$ comprises those features f satisfying
  (1) Sep(m_B, f) ≤ $r_B + r_j$
  (2) f also intersects the r_B-expansion of ~~TTS_j~~ NSS_j

Experimental Results

- Created challenging environments with several complex robots.





Summary of Experimental Results

Comparing with several sampling methods (PRM, RRT, EST, KPIECE) in open-source library OMPL.

- OMPL planners often have unsuccessful runs and have to time out even when there is a path.

- Our algorithms perform in real time, often much faster than OMPL planners, in addition to being able to report NO-PATH.

Video Demo

- Video is available at (link given in the paper) https://cs.nyu.edu/exact/gallery/complex/complex-robot-demo.mp4

- Code is available (link given in the paper): **Core Library** https://cs.nyu.edu/exact/core_pages/downloads.html

Conclusions

- We extended our SSS resolution-exact approach to challenging planning problems where no exact algorithms exist.

- Experiments show that our methods typically outperform OMPL sampling methods.

- Open Problems:
  (1) Optimal decomposition of m-gons into nice triangles?
  (2) Complex rigid robots in 3D?