POLYTECHNIC UNIVERSITY
Department of Computer and Information Science

Cartesian to Polar Coordinate Conversion: A Problem Solving Example

K. Ming Leung

Abstract: Our task is to write a computer program to convert the Cartesian coordinates of a point in two-dimension to polar coordinates. The resulting angle is measured in radians and can be negative.

Directory

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We assume that the Cartesian coordinates of an arbitrary point, x and y, in two-dimension are given. We need to write a computer program to convert them to polar coordinates. r and θ. We suppose that the angle θ is required to be in radians, and it is allowed to take on negative values when appropriate.

Computing the value for r is straightforward. It is given from Pythagorian theorem as

$$r = \sqrt{x^2 + y^2}.$$

The angle θ is considered as positive if it is measured counter-clockwise from the positive x-axis, and as negative if it is measured in a clockwise direction. Our remaining task is to compute it from x and y. It is clear that we need to use the `arctan` function in the mathematics software library to get θ:

$$\phi = \tan^{-1} u.$$

When using any library functions, we must know exactly the assumed domains and the corresponding ranges of these functions. In the case of the `arctan` function, it accepts any real value of the argument, so

u has the domain

$$-\infty < u < \infty.$$

The resulting angle has the following range

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2}.$$

When u is 0, ϕ is also 0. When u is positive, ϕ is also positive. As u increases so does ϕ. As u approaches $+\infty$, ϕ approaches $\frac{\pi}{2}$.

On the other hand, when u is negative, ϕ is also negative. As u decreases so does ϕ. As u approaches $-\infty$, ϕ approaches $-\frac{\pi}{2}$.

When the given point is in the first quadrant (Fig. 1), since both x and y are positive, and so the ratio $\frac{y}{x}$ is also positive. We see that the formula

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

correctly gives the polar angle since θ must be between 0 and $\frac{\pi}{2}$.

The next question is whether we can use this formula to compute θ if the point lies in the other quadrants.

When the given point is in the fourth quadrant (Fig. 2), x is positive but y is negative, and so the ratio $\frac{y}{x}$ is negative. Thus the

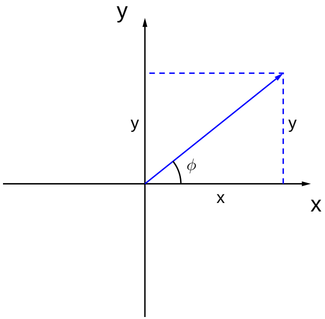

Figure 1: The point is located in the first quadrant.

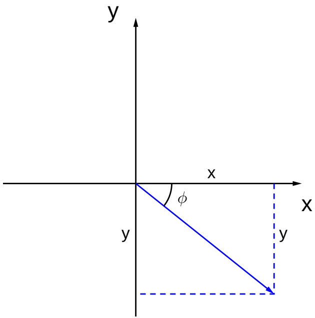

Figure 2: The point is located in the fourth quadrant.

angle ϕ (which may or may be the same as θ) computed using the formula

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

is negative and lies between $-\frac{\pi}{2}$ and 0. Since we accept negative angles, clearly this is an acceptable angle. Thus we have the formula for the polar angle

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

in the fourth quadrant as in the first quadrant.

When the given point is in the second quadrant (Fig. 3), x is negative but y is positive, and so the ratio $\frac{y}{x}$ is negative. Thus the angle ϕ (which may or may be the same as θ) computed using the formula

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

is negative and lies between $-\frac{\pi}{2}$ and 0. This is clearly not the polar angle we are looking for. The solution is to add π to this ϕ to obtain θ. Thus we have the formula for the polar angle

$$\theta = \pi + \tan^{-1}\left(\frac{y}{x}\right).$$

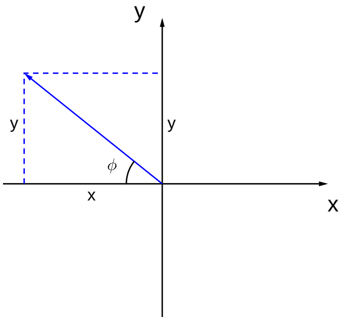

Figure 3: The point is located in the second quadrant.

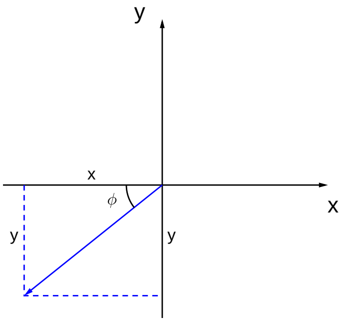

Figure 4: The point is located in the third quadrant.

Since ϕ is negative, θ is actually less than π (but larger than $\frac{\pi}{2}$).

Finally when the given point is in the third quadrant (Fig. 4), both x and y are negative, and so the ratio $\frac{y}{x}$ is positive. Thus the angle ϕ (which may or may be the same as θ) computed using the formula

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

is positive and lies between 0 and $\frac{\pi}{2}$. This is clearly not the polar angle we are looking for. The solution is to add π to this ϕ to obtain θ. Thus we have the formula for the polar angle

$$\theta = \pi + \tan^{-1}\left(\frac{y}{x}\right),$$

just as in the case of the third quadrant. The difference is ϕ is positive and so the angle is larger than π (but less than $\frac{3\pi}{2}$).

To summarize our results so far, we see that we must use Eqs. () for points to the right of the y-axis, and use Eqs. () for points to its left. But how about for point right on the y-axis?

Neither of these equations can be used since x is zero for those points, the arguments of the inverse tangent function would be infinite. Therefore we must consider the situation where the point is located on the y-axis. But this is an easy problem, since if $y > 0$ (the point is on the positive y-axis) then $\theta = \frac{\pi}{2}$, and if $y < 0$ (the point is on the negative y-axis) then $\theta = -\frac{\pi}{2}$, and if $y = 0$ (the point is located at the origin) then the angle is undefined. It can be set to an arbitrary angle, but we will follow customary practices and set it to 0.