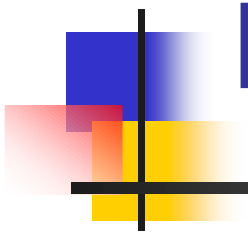


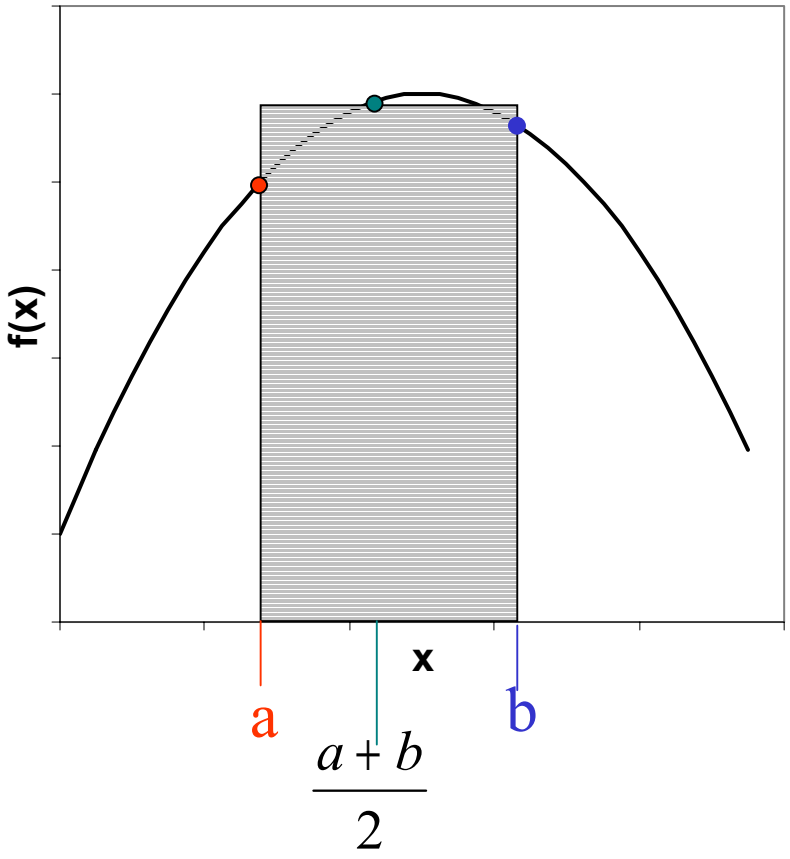
Numeric Integration

Integrating Data and Functions Numerically

Midpoint Rule

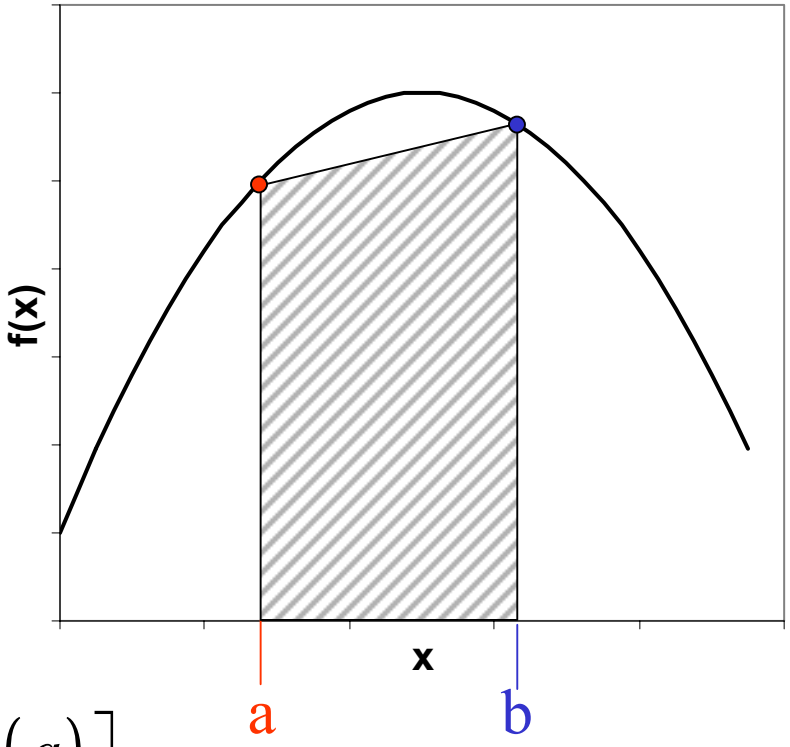
- Use the interval's ***midpoint*** to construct a rectangle. The integral is then approximated by the area of this rectangle.
- Overestimates the integral for concave down functions.
- Only evaluate the function at ***ONE*** point!!

$$\int_a^b f(x)dx \approx f\left(\frac{a+b}{2}\right)(b-a)$$



Trapezoidal Rule

- Uses the interval's ***endpoints*** to create a trapezoid. The integral is then approximated by the area of the trapezoid.
- Underestimates integral for concave down functions.
- Must evaluate the function at ***TWO*** points.



$$\int_a^b f(x)dx \approx (b-a)\left[\frac{f(b)+f(a)}{2}\right]$$

Simpson's Rule

- A combination of Midpoint and Trapezoid rules.
  - Combines one-third of the Midpoint rule with two-thirds of the Trapezoid rule.
  - Increases accuracy by balancing errors.
- Doesn't overshoot or undershoot consistently as Midpoint and Trapezoid rules do.
- Must evaluate the function at ***THREE*** points.

$$\int_a^b f(x)dx \approx \frac{(b-a)}{3}\left[\frac{f(a)+4f\left(\frac{a+b}{2}\right)+f(b)}{2}\right]$$

Quadrature

- Combine many divisions to increase accuracy of estimate for integral.
- Note that as number of divisions $\rightarrow\infty$ this becomes the *exact* solution for the integral

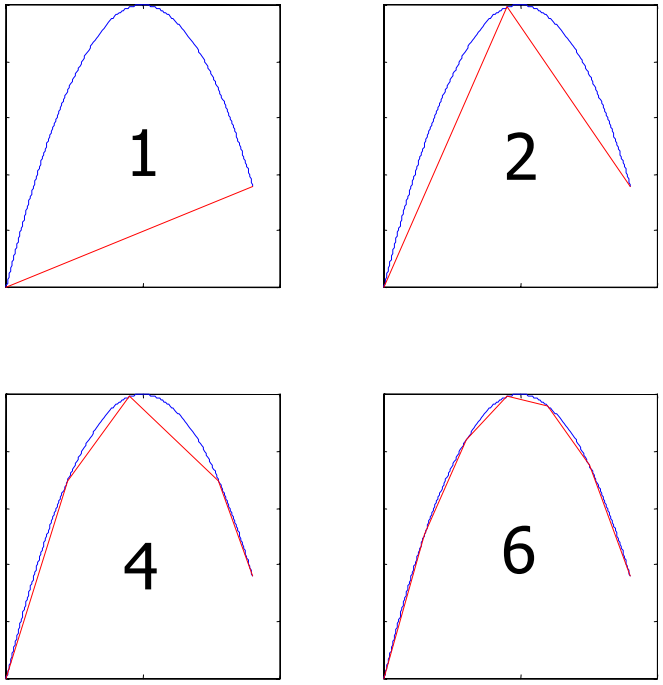
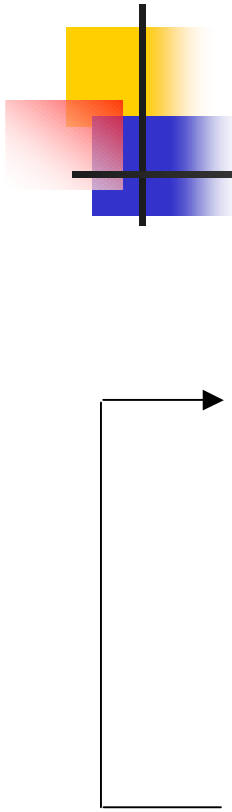


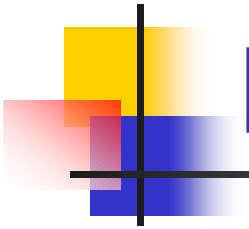
Illustration of composite Trapezoidal rule

An Algorithm for Numeric Integration



1. Given vectors of x and y
2. Compute Δx_i (it may not be constant)
3. Compute the integral over interval i using any method (ie Midpoint, Trapezoid, Simpson's)
4. Repeat 2 and 3 for each interval
5. Sum up over all intervals to get the approximation for the integral.

Numeric Integration of Discrete Data using MATLAB

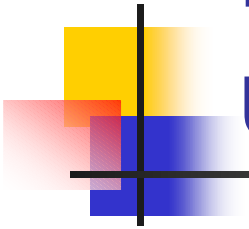


- TRAPZ(x,y)
  - x and y are vectors of equal length
  - Returns the approximate value for the integral of this data using the ***Trapezoidal rule.***
  - This is a simple function to write… Follow the algorithm presented previously!!!

Numeric Integration of Functions using MATLAB

- QUAD('F', a, b,tol,trace,p1,p2,…);
  - Uses ***Simpson's method*** with a sophisticated algorithm to compute numerical integrals
  - 'F' is the name of the function to be integrated
  - a and b are the limits of integration
  - tol specifies the tolerance (optional)
  - trace = 1 will plot the points where the function is evaluated (optional)
  - p1, p2, … are extra parameters that can be passed into the function (optional)

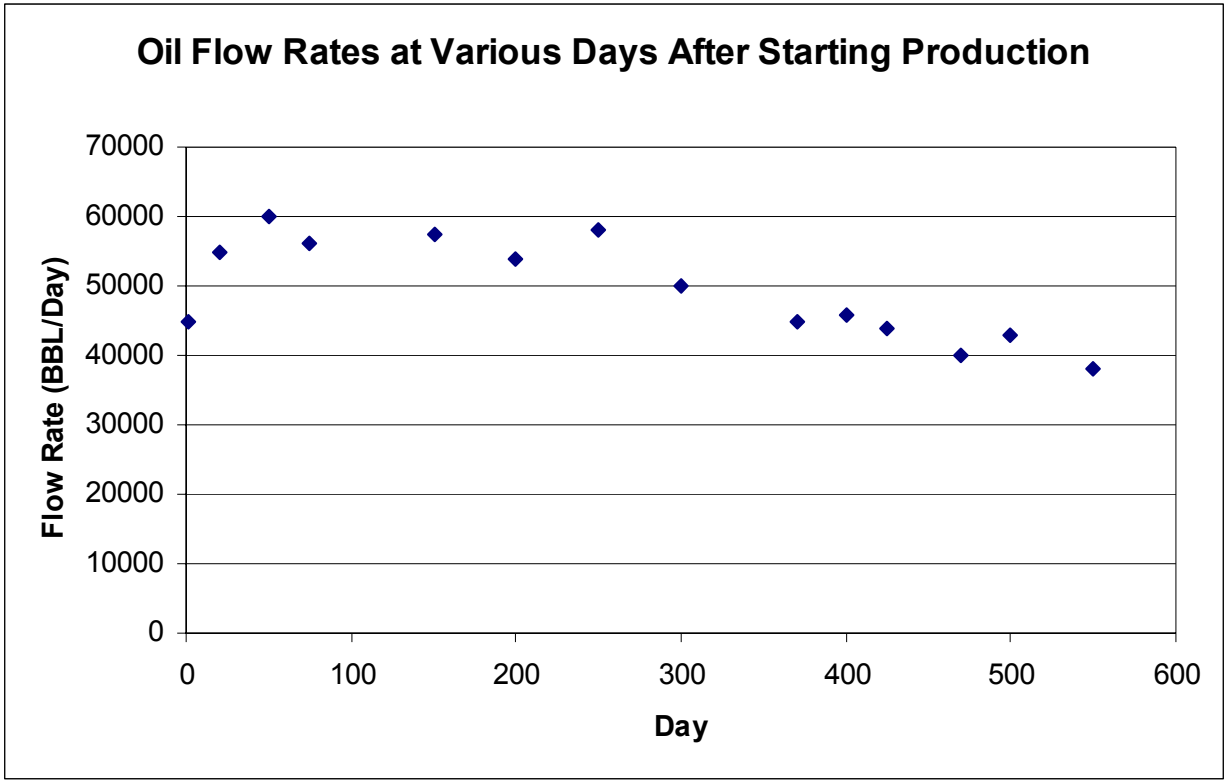
Numeric Integration of Functions using MATLAB (cont'd)



- QUAD8('F',a,b,tol,trace,p1,p2,…)
  - Higher order integration scheme (more accurate)
- DBLQUAD('F',INMIN,INMAX,OUTMIN,OUTMAX)
  - Evaluates Double integrals
  - INMIN is lower limit on inner integral, OUTMAX is upper limit on outer integral

Example: Oil Well Production

Day	Flow Rate (BBL/Day)
1	45000
20	55000
50	60000
75	56000
150	57500
200	54000
250	58000
300	50000
370	45000
400	45900
425	44000
470	40000
500	43000
550	38000



Estimate the total production from the well over this time period.