

# Computing $\mathbf{A}^{-1}\mathbf{B}$

Let  $\mathbf{A}$  be an  $n$  by  $n$  matrix, and  $\mathbf{x}$  and  $\mathbf{B}$  be both  $n$  by  $m$  matrices. Supposed matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given, and we want to solve the linear system of equations

$$\mathbf{AX} = \mathbf{B}$$

for  $\mathbf{X}$ . In component form this equation is

$$\sum_{j=1}^n A_{ij} X_{jk} = B_{ik}.$$

Therefore the problem is basically the same as the linear system

$$\mathbf{Ax} = \mathbf{b}$$

where  $\mathbf{x}$  and  $\mathbf{b}$  are  $n$ -vectors, except that we now have  $m$  copies of the problem, each having the same  $\mathbf{A}$  but having vectors  $\mathbf{x}$  and  $\mathbf{b}$  taken from each of the corresponding columns of  $\mathbf{X}$  and  $\mathbf{B}$  respectively.

Therefore LU factorization using Gaussian elimination can be used efficiently to find matrix  $\mathbf{X}$  without first explicitly finding  $\mathbf{A}^{-1}$  and then multiply by  $\mathbf{B}$  (a very inefficient process). We only need to perform LU

factorization of  $\mathbf{A}$  once, then forward and backward substitution can then be done for each columns of  $\mathbf{B}$  to obtain the corresponding column of  $\mathbf{X}$ .

