

ASSIGNMENT 2

Due February 17, 2004 (before start of class)

Problem 2

Use the following recursive relation

$$p_{n+1} = 2^{n+1} \sqrt{2 \left(1 - \sqrt{1 - \left(\frac{p_n}{2^{n+1}} \right)^2} \right)}, \quad \text{for } n = 1, 2, \dots, \quad (1)$$

to compute p_1, p_2, \dots, p_{35} . As n increases p_n is supposed to converge to the value of π (3.1415926535897932...). Start the iteration with $p_1 = 2\sqrt{2}$ and $n = 1$ to compute p_2 . Using p_2 one can then compute p_3 , etc.

1. Display your results in a two-column format. Column 1 for n and column 2 for p_n . Comment on the results that you get. Correct and insightful comments will be rewarded.
2. Notice that the above iteration using Eq. (1) is rather inefficient since it involves many unnecessary computations. See if you can change from p_n to some other variable to obtain a different iteration formula that can be iterated more efficiently. **Hint:** In the original formula one has to compute two square roots per iteration step. By squaring terms appropriately one of those square roots can be made to disappear. Some multiplications and divisions can also be avoided by suitable choice of the iteration variable.
3. Repeat the iteration using the new iteration variable. Display the results again in the two-column format. Comment on your results.
4. The iteration formula can be modified into a form which is suitable for high accuracy iteration. Repeat the iteration using this new form. Comment on your results. Make sure that your computation is done most efficiently, *i.e.* do not do any unnecessary computations. **Hint:** Notice the similarity between the formula in part 3 and the quadratic formula. The symptom and cure are exactly the same in both cases.