ASSIGNMENT 5 - Solution

b

Problem 5 - Solution

1. If each resistor has a resistance of $1\,\Omega$, and the constant voltage sources are given by $v_1^s = v_4^s = 15\,V$, we can solve the 4 by 4 linear system

$$\mathbf{R}\mathbf{i} = \mathbf{v}^s, \tag{1}$$

where

$$\mathbf{v} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 15 \end{bmatrix}, \tag{2}$$

using LU-factorization without pivoting solve for the current to get

$$\mathbf{i} = \begin{bmatrix} 11 \\ 7 \\ 10 \\ 12 \end{bmatrix}, \tag{3}$$

in amperes.

2. Next modify the circuit so that r_{14} has a new value of $6\,\Omega$ while everything else remains unchanged. That means that r_{14} is being increased by $5\,\Omega$, and so

$$\mathbf{R} \to \mathbf{R} + \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ -5 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

which we want to write as $\mathbf{R} - \mathbf{u}\mathbf{v}^T$. One possible choice is

$$\mathbf{u} = \begin{bmatrix} -5 \\ 0 \\ 5 \\ 0 \end{bmatrix} \tag{4}$$

and

$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}. \tag{5}$$

Then we solve

$$\mathbf{Rz} = \mathbf{u}$$

for $\mathbf{z}$, and solve

$$\mathbf{Ry} = \mathbf{v}^s$$

for $\mathbf{y}$. The new current is then given by

$$\mathbf{i} = \mathbf{y} + \frac{\mathbf{v}^T\mathbf{y}}{1 - \mathbf{v}^T\mathbf{z}}\mathbf{z}.$$

The computed result is

$$\mathbf{i} = \begin{bmatrix} 10.8 \\ 7.1 \\ 10.5 \\ 12.1 \end{bmatrix}. \tag{6}$$

3. This result agrees with the solution of the linear system using Matlab's linear system solver with the new $\mathbf{R}$ matrix.