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## A Brief MATLAB Tutorial

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#### Abstract

We present a brief MATLAB tutorial covering only the bare-minimum that a beginner needs to know in order to start writing programs in MATLAB.


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## 1. Introduction: What is MATLAB?

## MATLAB

1. integrates computation, visualization, and programming in an easy-to-use environment for the purpose of technical computing.
2 . is an interactive system whose basic data element is an array that does not require dimensioning.
2. incorporates many heavy-duty mathematical libraries such as the LAPACK and BLAS for matrix computation.
3. features a family of add-on application-specific solutions called toolboxes which consist of comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment.

## 2. The MATLAB System

The MATLAB system consists of the following five main parts.

1. The Development Environment is the set of tools and facilities that help you use MATLAB functions and files. Many of these
tools are graphical user interfaces. It includes the MATLAB desktop and Command Window, a command history, an editor and debugger, and browsers for viewing help, the workspace, files, and the search path.
2. The MATLAB Programming Language is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. It allows one to rapidly create quick and dirty throw-away programs, as well as complete large and complex application programs.
3. The MATLAB Mathematical Function Library is a vast collection of functions ranging from elementary ones like sum, sine, cosine, and complex arithmetic, to more sophisticated ones like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms. We will illustrate how to use the mathematical function when we deal with the MATLAB language.
4. MATLAB Graphics provide extensive facilities for displaying vectors and matrices as graphs, as well as annotating and print-
ing these graphs. It includes high-level functions for two- and three-dimensional data visualization, image processing, animation, and presentation graphics. It also includes low-level functions that allow you to fully customize the appearance of graphics as well as to build your own graphical user interfaces.
5. The MATLAB Application Program Interface (API) is a library that allows you to write C and Fortran programs that interact with MATLAB. It includes facilities for calling routines from MATLAB (dynamic linking), calling MATLAB as a computational engine, and for reading and writing MAT-files. We will not cover the API here.

### 2.1. The Development Environment

You work with MATLAB through its development environment.

- Starting and quitting MATLAB

To start MATLAB, double-click the MATLAB shortcut icon on your Windows desktop.

To end your MATLAB session, select Exit MATLAB from the File menu in the desktop, or close the main MATALAB window or type quit or exit in the Command Window.

## - MATLAB desktop

When you start MATLAB, the MATLAB desktop appears, containing tools (graphical user interfaces) for managing files, variables, and applications associated with MATLAB.

You can change the way your desktop looks by opening, closing, moving, and resizing the tools in it. Use the View menu to open or close the tools.

You can specify certain characteristics for the desktop tools by selecting Preferences from the File menu.

- MATLAB desktop tools

MATLAB has many desktop tools. The important ones are

- Command Window
- Editor
- Help Browser
- Current Directory Browser
- Workspace Browser

The most important tool is the Command Window. The command line is marked by the >> symbol. This is the place where the user enters variables and run functions and M-files. Each statement must be separated from each other by a comma, a semi-colon, or the return character. Output to the screen is suppressed if a statement is terminated with a semi-colon, otherwise results of a computation are displayed in the command window.

For example if the user types

```
>> radius = 2.3; area = pi*radius^2
```

the output is displayed in the command window as

```
area =
    16.6190
```

>>
where a new command line is shown after the output is displayed. Here pi is a predefined variable containing the value of pi.

Any input longer than one or two lines should be placed in a file which you can create using the MATLAB text editor. The file must end with the .m extension. Such a file is also call a script file. To run the script file, enter the name of the file (without the .m extension) on the command line.

Any file you want to run in MATLAB must reside in the current directory or in a directory that is on the search path. A quick way to view or change the current directory is by using the Current Directory field in the desktop toolbar.

### 2.2. The MATLAB Programming Language

Although MATLAB has other ways of storing both numeric and nonnumeric data, but in the beginning, it is best to think of everything as a matrix. Special meaning is sometimes attached to 1 -by- 1 matrices, which are scalars, and to matrices with only one row or column, which are vectors. MATLAB allows you to work with entire matrices
quickly and easily.
Other than one- and two-dimensional arrays MATLAB supports other data structures, such as multidimensional arrays, cell arrays, character and text data, structures and classes and objects.

It also has the if, elseif, and switch for controlling program flow and the for and while (but no do-while) for creating loops. It has practically every thing that any other modern programming language has. You will need to use the online help system or use the manual to look up the usages of various MATLAB build-in functions since the syntax are generally different than in other languages.

## - Entering Matrices

The best way for you to get started with MATLAB is to learn how to handle matrices. Start MATLAB and follow along with each example. You can enter matrices into MATLAB in several different ways:

Enter an explicit list of elements. Load matrices from external data files. Generate matrices using built-in functions. Create matrices with your own functions in M-files.

Start by entering Dürer's matrix as a list of its elements. You only
have to follow a few basic conventions: Separate the elements of a row with blanks or commas. Use a semicolon to indicate the end of each row. Surround the entire list of elements with square brackets. To enter Dürer's matrix, simply type on the command line

$$
A=\left[\begin{array}{llllllllllllll}
16 & 3 & 2 & 13 ; & 10 & 11 & 8 ; & 9 & 7 & 12 ; & 4 & 14 & 1
\end{array}\right]
$$

MATLAB displays the matrix you just entered.

$A=$| 16 | 3 | 2 | 13 |
| ---: | ---: | ---: | ---: |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Once you have entered the matrix, it is automatically remembered in the MATLAB workspace. You can refer to it simply as A.

- sum, transpose, and diag

You are probably already aware that the special properties of a magic square have to do with the various ways of summing its elements. If you take the sum along any row or column, or along either of the two
main diagonals, you will always get the same number. Let us verify that using MATLAB. The first statement to try is sum(A) MATLAB replies with

```
ans =
```

$$
\begin{array}{llll}
34 & 34 & 34 & 34
\end{array}
$$

When you do not specify an output variable, MATLAB uses the variable ans, short for answer, to store the results of a calculation. You have computed a row vector containing the sums of the columns of A. Sure enough, each of the columns has the same sum, the magic sum, 34. How about the row sums? MATLAB has a preference for working with the columns of a matrix, so the easiest way to get the row sums is to transpose the matrix, compute the column sums of the transpose, and then transpose the result. The transpose operation is denoted by an apostrophe or single quote, '. It flips a matrix about its main diagonal and it turns a row vector into a column vector. So A' produces

| ans $=$ |  |  |  |
| ---: | :--- | ---: | ---: | ---: |
| 16 | 5 | 9 | 4 |

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| 3 | 10 | 6 | 15 |
| ---: | ---: | ---: | ---: |
| 2 | 11 | 7 | 14 |
| 13 | 8 | 12 | 1 |

And
sum(A')'
produces a column vector containing the row sums
$\begin{aligned} & \text { ans }= \\ & 34 \\ & 34 \\ & 34 \\ & 34\end{aligned}$
The sum of the elements on the main diagonal is obtained with the sum and the diag functions. $\operatorname{diag}(\mathrm{A})$ produces

```
    ans =
```

        16
    10
    7
    1
    and
sum (diag (A))
produces
ans $=$
34
The other diagonal, the so-called antidiagonal, is not so important mathematically, so MATLAB does not have a ready-made function for it. But a function originally intended for use in graphics, fliplr, flips a matrix from left to right.

$$
\operatorname{sum}(\operatorname{diag}(f l i p l r(A)))
$$

ans $=$
34
You have verified that matrix A is indeed a magic square.

- Subscripts

The element in row $i$ and column $j$ of $A$ is denoted by $A(i, j)$. For example, $\mathrm{A}(4,2)$ is the number in the fourth row and second column.

For our magic square, $\mathrm{A}(4,2)$ is 15 . So to compute the sum of the elements in the fourth column of A , type

$$
\mathrm{A}(1,4)+\mathrm{A}(2,4)+\mathrm{A}(3,4)+\mathrm{A}(4,4)
$$

This produces

$$
\begin{array}{r}
\text { ans }= \\
34
\end{array}
$$

but is not the most elegant nor efficient way of summing a single column.

If you try to use the value of an element outside of the matrix, it is an error.

$$
t=A(4,5)
$$

Index exceeds matrix dimensions.
On the other hand, if you store a value in an element outside of the matrix, the size increases to accommodate the newcomer. For example

$$
\begin{aligned}
& X=A ; \\
& X(4,5)=17
\end{aligned}
$$

gives

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$X=$|  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |
| 16 | 3 | 2 | 13 | 0 |
| 5 | 10 | 11 | 8 | 0 |
| 9 | 6 | 7 | 12 | 0 |
| 4 | 15 | 14 | 1 | 17 |

Notice that all elements which have not been explicitly assigned a value, take on the default value of zero.

- The Colon Operator

The colon, :, is one of the most important MATLAB operators. It occurs in several different forms. The expression $1: 10$ is a row vector containing the integers from 1 to 10
1
2
3
4
5
6
7
8
9
10

To obtain nonunit spacing, specify an increment.
For example, 100:-7:50
is

| 100 | 93 | 86 | 79 | 72 | 65 | 58 | 51 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

and
0:pi/4:pi
is

$$
\begin{array}{lllll}
0 & 0.7854 & 1.5708 & 2.3562 & 3.1416
\end{array}
$$

Subscript expressions involving colons refer to portions of a matrix. $A(1: k, j)$ is the first $k$ elements of the $j$ th column of $A$. So sum $(A(1: 4,4))$ computes the sum of the fourth column. But there is a better way. The colon by itself refers to all the elements in a row or column of a matrix and the keyword end refers to the last row or column. So sum(A(: end)) computes the sum of the elements in the last column of $A$.
ans $=$
34
Why is the magic sum for a 4 -by- 4 square equal to 34 ? If the integers from 1 to 16 are sorted into four groups with equal sums, that sum must be $\operatorname{sum}(1: 16) / 4$ which, of course, is

```
ans =
```

34

- The magic Function

MATLAB actually has a built-in function that creates magic squares of almost any size. Not surprisingly, this function is named magic.

$$
\begin{aligned}
& \text { B = magic (4) } \\
& \text { B = }
\end{aligned}
$$

This matrix is almost the same as the one in the Dürer engraving and has all the same "magic" properties; the only difference is that the two middle columns are exchanged. To make this B into Dürer's A, swap the two middle columns.

$$
A=B\left(:,\left[\begin{array}{llll}
1 & 3 & 2 & 4
\end{array}\right]\right)
$$

This says, for each of the rows of matrix $B$, reorder the elements in the order $1,3,2,4$.

It produces
$\mathrm{A}=$

| 16 | 3 | 2 | 13 |
| ---: | ---: | ---: | ---: |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Why would Dürer go to the trouble of rearranging the columns when he could have used MATLAB ordering? No doubt he wanted to include the date of the engraving, 1514, at the bottom of his magic square.

## - Expressions

Like most other programming languages, MATLAB provides mathematical expressions, but unlike most programming languages, these expressions involve entire matrices. The building blocks of expressions are variables,numbers, operators and functions. MATLAB does not require any type declarations or dimension statements. When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes its contents and, if necessary, allocates new storage.

For example,

$$
\text { numStudents }=25
$$

creates a 1-by-1 matrix named numStudents and stores the value 25 in its single element. Variable names consist of a letter, followed by any number of letters, digits, or underscores. MATLAB uses only the first 31 characters of a variable name. MATLAB is case sensitive; it distinguishes between uppercase and lowercase letters. A and a are not the same variable. To view the values assigned to any variable, simply enter the variable name.

## - Numbers

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. Scientific notation uses the letter e to specify a power-of-ten scale factor. Imaginary numbers use either i or j as a suffix. Some examples of legal numbers are

| 3 | -99 | 0.0001 |
| :--- | :--- | :--- |
| 9.6397238 | $1.60210 \mathrm{e}-20$ | 6.02252 e 23 |

All numbers are stored internally using the long format specified by the IEEE floating-point standard. Floating-point numbers have a finite precision of roughly 16 significant decimal digits and a finite range of roughly $10^{-308}$ to $10^{+308}$.

## - Functions

MATLAB provides a large number of standard elementary mathematical functions, including abs, sqrt, exp, and sin. Taking the square root or logarithm of a negative number is not an error; the appropriate complex result is produced automatically. MATLAB also provides many more advanced mathematical functions, including Bessel and gamma functions. Most of these functions accept complex arguments. For a list of the elementary mathematical functions, type help elfun For a list of more advanced mathematical and matrix functions, type help specfun help elmat Some of the functions, like sqrt and sin, are built in. They are part of the MATLAB core so they are very efficient, but the computational details are not readily accessible. Other functions, like gamma and sinh, are implemented in M-files. You can

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see the code and even modify it if you want. Several special functions provide values of useful constants.

```
    pi 3.14159265...
    i Imaginary unit, \(\sqrt{-1}\)
    j Same as i
    eps Floating-point relative precision,
    Inf Infinity
NaN Not-a-number
```

Infinity is generated by dividing a nonzero value by zero, or by evaluating well defined mathematical expressions that overflow, i.e., exceed realmax. Not-a-number is generated by trying to evaluate expressions like $0 / 0$ or Inf-Inf that do not have well defined mathematical values. The function names are not reserved. It is possible to overwrite any of them with a new variable, such as eps $=1 . \mathrm{e}-6$ and then use that value in subsequent calculations. The original function can be restored with clear eps

- Examples of Expressions

You have already seen several examples of MATLAB expressions. Here are a few more examples, and the resulting values.

```
rho = (1+sqrt(5))/2
rho =
    1.6180
```

$a=a b s(3+4 i)$
$\mathrm{a}=$
5
z = sqrt(besselk(4/3,rho-i))
Z $=$
$0.3730+0.3214 i$
huge $=\exp (\log ($ realmax $))$
huge $=$
$1.7977 \mathrm{e}+308$

```
toobig = pi*huge
toobig =
    Inf
```

- Generating Matrices

MATLAB provides four functions that generate basic matrices.

| zeros | All zeros |
| ---: | :--- |
| ones | All ones |
| rand | Uniformly distributed random elements |
| randn | Normally distributed random elements |

Here are some examples.

| $Z=$ | $\operatorname{zeros}(2,4)$ |  |  |
| ---: | :--- | ---: | :--- |
| $Z=$ |  |  |  |
|  | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |

$$
F=5 * \text { ones }(3,3)
$$

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- M-Files

You can create your own matrices using M-files, which are text files containing MATLAB code. Use the MATLAB Editor or another text
editor to create a file containing the same statements you would type at the MATLAB command line. Save the file under a name that ends in .m. For example, create a file containing these five lines.

| $\mathrm{A}=$ [ |  |  |  |
| :---: | :---: | :---: | :---: |
| 16.0 | 3.0 | 2.0 | 13.0 |
| 5.0 | 10.0 | 11.0 | 8.0 |
| 9.0 | 6.0 | 7.0 | 12.0 |
| 4.0 | 15.0 | 14.0 | 1.0 |

Store the file under the name magik.m. Then the statement magik reads the file and creates a variable, A, containing our example matrix.

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, [ ], is the concatenation operator. For an example, start with the 4 -by- 4 magic square, $A$, and form $B=[A \quad A+32 ; A+48 A+16]$ The result is an 8 -by- 8 matrix, obtained by joining the four submatrices.

$$
B=
$$

| 16 | 3 | 2 | 13 | 48 | 35 | 34 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| 5 | 10 | 11 | 8 | 37 | 42 | 43 | 40 |
| ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- |
| 9 | 6 | 7 | 12 | 41 | 38 | 39 | 44 |
| 4 | 15 | 14 | 1 | 36 | 47 | 46 | 33 |
| 64 | 51 | 50 | 61 | 32 | 19 | 18 | 29 |
| 53 | 58 | 59 | 56 | 21 | 26 | 27 | 24 |
| 57 | 54 | 55 | 60 | 25 | 22 | 23 | 28 |
| 52 | 63 | 62 | 49 | 20 | 31 | 30 | 17 |

This matrix is halfway to being another magic square. Its elements are a rearrangement of the integers 1:64. Its column sums are the correct value for an 8 -by- 8 magic square. sum (B)
ans =

$$
\begin{array}{llllllll}
260 & 260 & 260 & 260 & 260 & 260 & 260 & 260
\end{array}
$$

But its row sums, sum (B') ', are not all the same. Further manipulation is necessary to make this a valid 8 -by- 8 magic square.

Deleting Rows and Columns
You can delete rows and columns from a matrix using just a pair of square brackets. Start with $\mathrm{X}=\mathrm{A}$; Then, to delete the second column of $X$, use $X(:, 2)=[]$. This changes $X$ to

| $X=$ |  |  |
| ---: | ---: | ---: |
| 16 | 2 | 13 |
| 5 | 11 | 8 |
| 9 | 7 | 12 |
| 4 | 14 | 1 |

If you delete a single element from a matrix, the result is not a matrix anymore. So, expressions like $\mathrm{X}(1,2)=[$ ] result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector.

So

$$
x(2: 2: 10)=[]
$$

results in

$$
\mathrm{X}=
$$

| 16 | 9 | 2 | 7 | 13 | 12 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

- Linear Algebra

Informally, the terms matrix and array are often used interchangeably. More precisely, a matrix is a two-dimensional numeric array that rep-
resents a linear transformation. The mathematical operations defined on matrices are the subject of linear algebra. Dürer's magic square

$A=$| 16 | 3 | 2 | 13 |
| ---: | ---: | ---: | ---: |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

provides several examples that give a taste of MATLAB matrix operations. You have already seen the matrix transpose, A'. Adding a matrix to its transpose produces a symmetric matrix. A + A'

| ans $=$ |  |  |  |
| ---: | ---: | ---: | ---: |
| 32 | 8 | 11 | 17 |
| 8 | 20 | 17 | 23 |
| 11 | 17 | 14 | 26 |
| 17 | 23 | 26 | 2 |

The multiplication symbol, *, denotes the matrix multiplication involving inner products between rows and columns. Multiplying the transpose of a matrix by the original matrix also produces a symmet-

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ric matrix. A' $* \mathrm{~A}$

ans |  |  |  |  |
| ---: | :--- | :--- | :--- |
| 378 | 212 | 206 | 360 |
| 212 | 370 | 368 | 206 |
| 206 | 368 | 370 | 212 |
| 360 | 206 | 212 | 378 |

The determinant of this particular matrix happens to be zero, indicating that the matrix is singular. $d=\operatorname{det}(A)$

$$
d=
$$

The reduced row echelon form of A is not the identity.

$$
R=\operatorname{rref}(A)
$$

$R=$| 1 | 0 | 0 | 1 |
| ---: | ---: | ---: | ---: |
| 0 | 1 | 0 | -3 |
| 0 | 0 | 1 | 3 |
| 0 | 0 | 0 | 0 |

Since the matrix is singular, it does not have an inverse. If you try to compute the inverse with $\mathrm{X}=\operatorname{inv}(\mathrm{A})$ you will get a warning message Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND $=1.175530 \mathrm{e}-017$. Roundoff error has prevented the matrix inversion algorithm from detecting exact singularity. But the value of rcond, which stands for reciprocal condition estimate, is on the order of eps, the floating-point relative precision, so the computed inverse is unlikely to be of much use. The eigenvalues of the magic square are interesting.

$$
\begin{aligned}
& e=\operatorname{eig}(A) \\
& e= \\
& 34.0000 \\
& 8.0000 \\
& 0.0000 \\
& -8.0000
\end{aligned}
$$

One of the eigenvalues is zero, which is another consequence of singularity. The largest eigenvalue is 34 , the magic sum. That is because

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the vector of all ones is an eigenvector.

$$
\mathrm{v}=\operatorname{ones}(4,1)
$$

$$
\mathrm{v}=
$$

$$
1
$$

$$
1
$$

$$
1
$$

$$
1
$$

A*V

```
ans =
```

34
34
34
34

## - Arrays

When they are taken away from the world of linear algebra, matrices become two-dimensional numeric arrays. Arithmetic operations on arrays are done element-by-element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations. The list of operators includes

+ Addition
- Subtraction
.* Element-by-element multiplication
* Matrix multiplication
./ Element-by-element division
Unconjugated array transpose
If the Dürer magic square is multiplied by itself with array multiplication A. $*$ A the result is an array containing the squares of the integers from 1 to 16 , in an unusual order.

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ans $=$|  |  |  |  |
| ---: | ---: | ---: | ---: |
| 256 | 9 | 4 | 169 |
| 25 | 100 | 121 | 64 |
| 81 | 36 | 49 | 144 |
| 16 | 225 | 196 | 1 |

- Building Matrices and Tables

Array operations are useful for building tables. Suppose n is the column vector

$$
\mathrm{n}=(0: 9)^{\prime} ;
$$

Then

$$
\text { pows }=\left[\begin{array}{lll}
n & n . \wedge 2 & 2 . \wedge n
\end{array}\right]
$$

builds a table of squares and powers of 2 .

| pows $=$ |  |  |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 1 | 1 | 2 |
| 2 | 4 | 4 |

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| 3 | 9 | 8 |
| ---: | ---: | ---: |
| 4 | 16 | 16 |
| 5 | 25 | 32 |
| 6 | 36 | 64 |
| 7 | 49 | 128 |
| 8 | 64 | 256 |
| 9 | 81 | 512 |

The elementary math functions operate on arrays element by element. So

```
format short g
x = (1:0.1:2)';
logs = [x log10(x)]
```

builds a table of logarithms.

| $\operatorname{logs}=$ | 0 |
| ---: | ---: |
| 1.0 | 0.04139 |
| 1.1 | 0.07918 |
| 1.2 | 0.11394 |
| 1.3 | 0.139 |


| 1.4 | 0.14613 |
| :--- | :--- |
| 1.5 | 0.17609 |
| 1.6 | 0.20412 |
| 1.7 | 0.23045 |
| 1.8 | 0.25527 |
| 1.9 | 0.27875 |
| 2.0 | 0.30103 |

- Scalar Expansion

Matrices and scalars can be combined in several different ways. For example, a scalar is subtracted from a matrix by subtracting it from each element. The average value of the elements in our magic square is 8.5 , so $B=A-8.5$ forms a matrix whose column sums are zero.

$$
\begin{array}{lrrr}
\text { B }= & & & \\
7.5 & -5.5 & -6.5 & 4.5 \\
-3.5 & 1.5 & 2.5 & -0.5 \\
0.5 & -2.5 & -1.5 & 3.5 \\
-4.5 & 6.5 & 5.5 & -7.5
\end{array}
$$

## sum (B)

ans =
$\begin{array}{llll}0 & 0 & 0 & 0\end{array}$
With scalar expansion, MATLAB assigns a specified scalar to all indices in a range. For example, $B(1: 2,2: 3)=0$ zeroes out a portion of B.

$B=$|  |  |  |  |
| ---: | ---: | ---: | ---: |
| 7.5 | 0 | 0 | 4.5 |
| -3.5 | 0 | 0 | -0.5 |
| 0.5 | -2.5 | -1.5 | 3.5 |
| -4.5 | 6.5 | 5.5 | -7.5 |

- Logical Subscripting

The logical vectors created from logical and relational operations can be used to reference subarrays.

Suppose X is an ordinary matrix and L is a matrix of the same size that is the result of some logical operation. Then $\mathrm{X}(\mathrm{L})$ specifies the elements of X where the elements of L are nonzero. This kind of

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subscripting can be done in one step by specifying the logical operation as the subscripting expression. Suppose you have the following set of data.

$$
\begin{array}{rlllllllllllllllllllll}
\mathrm{x}= \\
2.1 & 1.7 & 1.6 & 1.5 & \mathrm{NaN} & 1.9 & 1.8 & 1.5 & 5.1 & 1.8 & 1.4 & 2.2 & 1.6 & 1.8
\end{array}
$$

The NaN is a marker for a missing observation, such as a failure to respond to an item on a questionnaire. To remove the missing data with logical indexing, use finite(x), which is true for all finite numerical values and false for NaN and Inf.

```
x = x(finite(x))
x =
    2.1 1.7 1.6 1.5 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

Now there is one observation, 5.1, which seems to be very different from the others. It is an outlier. The following statement removes outliers, in this case those elements more than three standard deviations from the mean.

$$
\begin{aligned}
& x=x(\operatorname{abs}(x-\operatorname{mean}(x))<=3 * \operatorname{std}(x)) \\
& x=
\end{aligned}
$$

For another example, highlight the location of the prime numbers in Dürer's magic square by using logical indexing and scalar expansion to set the nonprimes.

## - The find Function

The find function determines the indices of array elements that meet a given logical condition. In its simplest form, find returns a column vector of indices. Transpose that vector to obtain a row vector of indices. For example, $\mathrm{k}=$ find(isprime(A))' picks out the locations, using one-dimensional indexing, of the primes in the magic square.

$$
\mathrm{k}=
$$

$\begin{array}{llllll}2 & 5 & 9 & 10 & 11 & 13\end{array}$
Display those primes, as a row vector in the order determined by k , with A(k)

| ans $=$ |  |  |  |  |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 3 | 2 | 11 | 7 | 13 |

When you use k as a left-hand-side index in an assignment statement, the matrix structure is preserved.

$$
\mathrm{A}(\mathrm{k})=\mathrm{NaN}
$$

| $\mathrm{A}=$ |  |  |  |
| ---: | ---: | ---: | ---: |
| 16 | NaN | NaN | NaN |
| NaN | 10 | NaN | 8 |
| 9 | 6 | NaN | 12 |
| 4 | 15 | 14 | 1 |

- Controlling Command Window Input and Output

The format Function can be used to control the appearance of numeric format of the values displayed by MATLAB. The function affects only how numbers are displayed, not how MATLAB computes or saves them. The MATLAB statement is
format choice
where choice should be replaced by one of the following formatting choices. For example, given a vector x with components of different
magnitudes

$$
x=\left[\begin{array}{ll}
4 / 3 & 1.2345 e-6
\end{array}\right]
$$

here are the different format choices, together with the resulting output produced from them
choices

| short | 1.3333 | 0.0000 |
| ---: | ---: | ---: |
| short e | $1.3333 \mathrm{e}+000$ | $1.2345 \mathrm{e}-006$ |
| short g | 1.3333 | $1.2345 \mathrm{e}-006$ |
| long | 1.33333333333333 | 0.00000123450000 |
| long e | $1.33333333333333 \mathrm{e}+000$ | $1.234500000000000 \mathrm{e}-006$ |
| long g | 1.33333333333333 | $1.2345 \mathrm{e}-006$ |
| bank | 1.33 | 0.00 |
| rat | $4 / 3$ | $1 / 810045$ |
| hex | $3 f f 5555555555555$ | 3eb4b6231abfd271 |

If the largest element of a matrix is larger than $10^{3}$ or smaller than $10^{-3}$, MATLAB applies a common scale factor for the short and long formats. In addition to the format functions shown above format
compact suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. If you want more control over the output format, use the sprintf and fprintf functions.

## - Suppressing Output

If you simply type a statement and press Return or Enter, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation but does not display any output. This is particularly useful when you generate large matrices. For example, A $=\operatorname{magic}(100)$;

## - Entering Long Statements

If a statement does not fit on one line, use an ellipsis (three periods), ..., followed by Return or Enter to indicate that the statement continues on the next line.

For example,

$$
\begin{aligned}
\mathrm{s}= & 1-1 / 2+1 / 3-1 / 4+1 / 5-1 / 6+1 / 7 \ldots \\
& -1 / 8+1 / 9-1 / 10+1 / 11-1 / 12
\end{aligned}
$$

Blank spaces around the $=,+$, and - signs are optional, but they improve readability.

## - Vectorization

To obtain the most speed out of MATLAB, it's important to vectorize the algorithms in your M-files. Where other programming languages might use for or DO loops, MATLAB can use vector or matrix operations. A simple example involves creating a table of logarithms.

$$
\begin{aligned}
& \mathrm{x}=.01 ; \\
& \text { for } \mathrm{k}=1: 10000 \\
& \quad \mathrm{y}(\mathrm{k})=\log 10(\mathrm{x}) ; \\
& \quad \mathrm{x}=\mathrm{x}+.01
\end{aligned}
$$

end
A vectorized version of the same code is

$$
\begin{aligned}
& x=.01: .01: 100 ; \\
& y=\log 10(x) ;
\end{aligned}
$$

For more complicated code, vectorization options are not always so obvious. When speed is important, however, you should always look
for ways to vectorize your algorithms but eliminating loops.

### 2.3. MATLAB Graphics

MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as annotating and printing these graphs.

## - Simple graphs

The plot function has different forms, depending on the input arguments. If y is a vector, plot(y) produces a piecewise linear graph of the elements of $y$ versus the index of the elements of $y$. If you specify two vectors as arguments, plot(x,y) produces a graph of y versus x . For example, these statements use the colon operator to create a vector of x values ranging from zero to $2 \pi$, compute the sine of these values, and plot the result.

$$
\begin{aligned}
& \mathrm{x}=0: \mathrm{pi} / 100: 2 * \mathrm{pi} ; \\
& \mathrm{y}=\sin (\mathrm{x}) \\
& \mathrm{plot}(\mathrm{x}, \mathrm{y})
\end{aligned}
$$

Now label the axes and add a title. The characters pi create the symbol $\pi$.

```
xlabel('x = 0:2\pi')
ylabel('sin(x)')
title('Plot of the Sine Function','FontSize',12)
```

- Multiple Data Sets in One Graph

Multiple $x-y$ pair arguments create multiple graphs with a single call to plot. MATLAB automatically cycles through a predefined (but user settable) list of colors to allow discrimination among sets of data. For example, these statements plot three related functions of $x$, each curve in a separate distinguishing color.

$$
\begin{aligned}
& y 2=\sin (x-.25) ; \\
& y 3=\sin (x-.5) ; \\
& \text { plot }(x, y, x, y 2, x, y 3)
\end{aligned}
$$

The legend command provides an easy way to identify the individual plots.

$$
\text { legend('sin( } \left.x)^{\prime}, ' \sin (x-.25)^{\prime}, ' \sin (x-.5)^{\prime}\right)
$$

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Most of the attributes of a plot, such as color, line style, line thickness and font size, can be edited and modified interactively using tools that are available in the figure window.

## References

[1] Most the materials here are adopted from MATLAB's online documentation, which can be accessed within MATLAB by selecting MATLAB Help from the Help menu. The documentation can also be accessed through the MATLAB website.
[2] K. Sigmon and T. A. Davis, MATLAB Primer, Sixth Edition, Chapman and Hall/CRC, 2002.
[3] D. J. Higham and N. J. Higham, MATLAB Guide, SIAM, 2000.
[4] D. C. Hanselman and B. Littlefield, Mastering MATLAB 6, A Comprehensive Tutorial and Reference, Prentice-Hall, 2000.

