ASSIGNMENT 1

Due September 21, 2004, before 11:00 am

Problem 1

This problem is a modified version of Exercise 4.3 in Heath's book. Let a 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}.$$

Your answers to the following questions should be numeric and specific to the particular matrix, not just the general definitions.

(a) What is the characteristic polynomial of $\mathbf{A}$? Write out the polynomial with its coefficients.

(b) What are the roots of the characteristic polynomial of $\mathbf{A}$?

(c) What are the eigenvalues of $\mathbf{A}$?

(d) What are the eigenvectors of $\mathbf{A}$?

(e) Perform two iterations of normalized power iteration on A, using $\mathbf{x}_0 = [1 \;\; 1]^T$ as starting vector. Use the infinite norm for normalization at each iteration.

(f) To what eigenvector of $\mathbf{A}$ will normalized power iteration ultimately converge? Use the infinite norm for normalization and a starting vector $\mathbf{x}_0 = [1 \;\; 1]^T$. Set a tolerance of 1×10^{-4} and show the output results for the eigenvalue and eigenvector for each iteration.

(g) What eigenvalue estimate is given by the Rayleigh quotient, using the vector $\mathbf{x} = [1 \;\; 1]^T$?

(h) To what eigenvector of $\mathbf{A}$ would inverse iteration ultimately converge? Use $\mathbf{x}_0 = [1 \;\; 1]^T$ as starting vector. Show the output result for the eigenvector for each iteration.

(i) What eigenvalue of $\mathbf{A}$ would be obtained if inverse iteration were used with shift $\sigma = 2$? Use $\mathbf{x}_0 = [1 \;\; 1]^T$ as starting vector. Show the output result for the eigenvector for each iteration. Please note that the Matlab function program ShiftedInverseIterationF.m posted at my website for doing shifted inverse iteration unfortunately contains errors. You need to correct those errors first before you use it to solve the present problem.