ASSIGNMENT 3

Due October 21, 2004, before 11:00 am

Problem 3

This problem is essentially the same as Computer Problem 6.10 on p. 304 of Heath's book.

Let $\mathbf{A}$ be an $n \times n$ real symmetric matrix with eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. (Note that λ_n is the dominant eigenvalue and λ_1 is the least dominant eigenvalue.) It can be shown that the critical points of the Rayleigh quotient (see Section 4.5.3) are eigenvectors of $\mathbf{A}$, in particular

$$\lambda_1 = \min_{\mathbf{x} \neq \mathbf{0}} \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}}$$

$$\lambda_n = \max_{\mathbf{x} \neq \mathbf{0}} \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}}$$

with the minimum and maximum occurring at the corresponding eigenvectors. Thus we can in principle compute the extreme eigenvalues and corresponding eigenvectors of $\mathbf{A}$ using any suitable method for optimization.

(a) Use the steepest descent method for unconstraint optimization to compute the extreme eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{A}_1 = \begin{bmatrix} 6 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Is the solution unique in each case? Why or why not?

(b) The foregoing characterization of λ_1 and λ_n remains valid if we restrict the vector $\mathbf{x}$ to be normalized by taking $\mathbf{x}^T \mathbf{x} = 1$. Repeat part a, but apply the steepest descent method for constrained optimization to impose this normalization constraint.