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CS 6673 spring 2009

Assignment 4, Solution

Initial weights and biases:

$$W^{(1)} = 1 \qquad b^{(1)} = -2, \qquad W^{(2)} = 1 \qquad b^{(2)} = 1.$$

We need to compute the derivatives:

$$f^{1)}(n) = n^2, \quad \Rightarrow \quad \dot{f}^{1)}(n) = 2n$$

$$f^{2)}(n) = \frac{1}{n}, \quad \Rightarrow \quad \dot{f}^{2)}(n) = -\frac{1}{n^2}.$$

For $x = a^{(0)} = 1$ and $t = 1$ we want to perform one iteration of the backpropagation algorithm with $\alpha = 1$.

$$n^{(1)} = a^{(0)}W^{(1)} + b^{(1)} = 1 \times 1 + (-2) = -1.$$
$$a^{(1)} = f^{(1)}(n^{(1)}) = (-1)^2 = 1.$$

$$n^{(2)} = a^{(1)}W^{(2)} + b^{(2)} = 1 \times 1 + 1 = 2.$$
$$a^{(2)} = f^{(2)}(n^{(2)}) = \frac{1}{2}.$$

Compute the sensitivities:

$$s^{(2)} = 2(a^{(2)} - t)\dot{f}^{(2)}(n^{(2)}) = 2 \times \left(\frac{1}{2} - 1\right) \times \left(-\frac{1}{2^2}\right) = \frac{1}{4}.$$

$$s^{(1)} = \dot{f}^{(1)}(n^{(1)})W^{(2)}s^{(2)} = 2 \times (-1) \times 1 \times \frac{1}{4} = -\frac{1}{2}.$$

Update the weights and biases:

$$W^{(1)} = 1 - \alpha a^{(0)}s^{(1)} = 1 - 1 \times 1 \times \left(-\frac{1}{2}\right) = \frac{3}{2}.$$

$$b^{(1)} = -2 - \alpha s^{(1)} = -2 - 1 \times \left(-\frac{1}{2}\right) = -\frac{3}{2}.$$

$$W^{(2)} = 1 - \alpha a^{(1)} s^{(2)} = 1 - 1 \times 1 \times \left(-\frac{1}{4}\right) = \frac{3}{4}.$$

$$b^{(2)} = -2 - \alpha s^{(2)} = 1 - 1 \times \left(-\frac{1}{4}\right) = -\frac{3}{4}.$$