

ASSIGNMENT 4

February March 6, 2003 (before 3:00 pm)

Homework 4: Evaluating a D-Dimensional Integral Using the Sample-Mean Monte Carlo Method

Use the Sample-Mean Monte Carlo method to evaluate the following D-dimensional integral:

$$I = \int_0^1 \dots \int_0^1 \sin^2 \left(\frac{\pi}{4} (x_1 + x_2 + \dots + x_D) \right) dx_1 dx_2 \dots dx_D.$$

You can compute the actual errors in the simulation since the exact analytical result for I is known:

$$I = \frac{1}{2} - \frac{2^{3D/2}}{2\pi^D} \cos \left(\frac{D\pi}{4} \right)$$

to. Use $D = 5$ for this problem.

1. Use 100,000 points in your simulation and compare the most probable error that you estimate from the simulation data with the actual error.
2. Next investigate the dependence of the error with the number of points used in the simulation. Use $N = 100, 1,000, 10,000,$ and $100,000$. For each value of N repeat the calculation 20 times and compute the average of the errors. Plot the average errors against the negative of the logarithm of N . How does the average errors depend on N ?