ASSIGNMENT 6

April 29, 2003 (before 6:00 pm)

Homework 6: Quasi-Monte Carlo Simulation using the Halton Sequence

In homework 4 we computed the multi-dimension integral

$$I = \int_0^1 \ldots \int_0^1 \sin^2\left(\frac{\pi}{4}(x_1 + x_2 + \ldots + x_D)\right) dx_1 dx_2 \ldots dx_D.$$

for $D = 5$ using the Sample-Mean Monte Carlo method. We recall that the exact analytical result for I is known:

$$I = \frac{1}{2} - \frac{2^{3D/2}}{2\pi^D} \cos\left(\frac{D\pi}{4}\right)$$

and so we can actually measure the error in any computed result.

In this problem we want to redo this integral by a quasi-Monte Carlo method using the Halton sequence.

1. You need to be able to generate the Halton sequence in 5-dimensions using prime numbers $2, 3, 5, 7$ and 11. You can seek help by using the Internet if necessary.
2. Check your numbers to make sure that they are the same as the ones I have.
3. No need to print out the numbers that you generate. Only show me H_j for $j = 12345, 12346$ and 12347.
4. Using $10,000$ points to compute the mean and the variance in each of the 5 dimensions.
5. Finally use the Halton sequence to compute the value of the above integral and compare with the result obtained using the Sample-Mean Monte Carlo method.
6. Plot the logarithm of the error versus the negative of the logarithm of the number of point used, N. Of course you need to repeat the simulation using different N each time. What is the dependence of the error on N?