

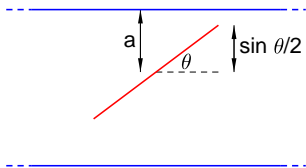
## Buffon's Needle

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**Abstract:** Buffon's needle is another way in which the value of  $\pi$  can be computed by simulating the dropping of a needle onto a floor with equal distant parallel cracks.

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Buffon's Needle[1, 2] is one of the oldest problems in the field of geometrical probability. It was first stated by the French naturalist and mathematician, Comte de Buffon (1707-1788) in 1777. The original version of it goes as follows. Suppose you have a floor made of long wooden planks each having a width equal to 1 unit of length. A large number,  $N$ , of plastic coffee stirrers (originally used needles), each of length also of 1 unit, are dropped randomly onto the floor. If  $N'$  is the total number of those stirrers which either touch or cross the cracks between any two adjacent planks, then the value of  $\pi$  can be estimated as  $2N/N'$ .

Let us see why that is so. We can assume that the planks are all lying parallel to the  $x$ -axis. Let  $a$  be the perpendicular distance



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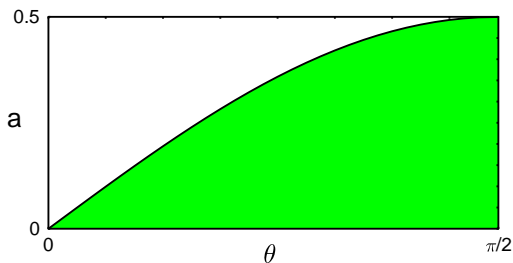
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between the center of a straw and a nearest crack, and let  $\theta$  be the angle that the straw makes with the  $x$ -axis. It is clear that if  $\sin \theta/2 \geq a$  then the straw either touches or crosses its nearest crack. We want to know the probability that that happens. The random variables are  $a$  and  $\theta$ . We assume that there is no correlation between the variables. Thus  $a$  is distributed randomly and uniformly in  $[0, 1/2]$ , and  $\theta$  is distributed randomly and uniformly in  $[0, \pi/2]$ . Notice that because of the symmetry of the problem there is no need to let  $\theta$  go all the way to  $\pi$ . It is much more efficient not to do that.

Next we consider the following graph where  $\theta$  lies on the horizontal axis and  $a$  lies on the vertical axis. In this graph we also plot the curve  $\frac{1}{2} \sin \theta$  versus  $\theta$ . Each point inside the rectangle bounded by  $\theta \in [0, \pi/2]$  and  $a \in [0, 1/2]$  represents a possible position of the center of a stirrer and its orientation. Points below the curve represent straws which either touch or cross a crack. Therefore the probability for that to happen is given by the ratio of the area under the curve to the area of the rectangle. That ratio must also be given by  $N'/N$ . Thus we

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have

$$\frac{N'}{N} = \frac{\int_0^{\pi/2} d\theta \frac{1}{2} \sin \theta}{\frac{1}{2} \frac{\pi}{2}} = \frac{\frac{1}{2} \cos \theta \Big|_0^{\pi/2}}{\frac{\pi}{4}} = \frac{2}{\pi},$$

and so we can compute the value of  $\pi$  from the simulation data:

$$\pi = \frac{2N}{N'}.$$

There are two other possibilities for the relationship between the



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length of the needles and the distance between the cracks. A good discussion of these can be found in Schroeder,[6]. The situation in which the distance between the cracks is greater than the length of the needle is an extension of the above explanation and the probability of a hit is  $2L/K\pi$  where  $L$  is the length of the needle and  $K$  is the distance between the cracks. The situation in which the needle is longer than the distance between the cracks leads to a slightly more complicated formula than the one given above for  $\pi$ .

The problem can be extended to a "needle" in the shape of a convex polygon with generalized diameter less than  $a$ . The probability that the boundary of the polygon will intersect one of the cracks is related to the perimeter of the polygon.[4, 3, 7] A further generalization obtained by throwing a needle on a board ruled with two sets of equally spaced and mutually perpendicular lines is called the Buffon-Laplace needle problem.[4, 3]

## References

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- [5] W. Cheney and D. Kincaid, *Numerical Mathematics and Computing*, 2nd Ed., (Brooks/Cole, 1985), pp. 354-354.
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